4

2. The Problem of Quantum Statistics

In classical ~~nineteenth century~~ atomic physics the strongest version
of PII, with attributes restricted to
monadic intrinsic properties, is clearly
false for the atoms ~~and molecules~~. The weak
version is forever true, if we allow
an impenetrability assumption (IA) to the
effect that ~~two atoms~~ distinct atoms
can never occupy the same location
in space. We want to begin our
discussion of quantum physics by
considering an argument to the
effect that quantum particles cannot
be ~~individuated~~ at all regarded as
individuals at all. ~~Hence the problem~~
If this were the case the problem of
how they are individuated simply
would not arise. PII would be
not be either true or false but simply
~~is apt~~ inapplicable.

The argument runs like this. Consider
the problem of distinguishing two
quantum particles # initially supposed
to be individuals # and labelled
1 and 2, among two possible pure
quantum states $|a^n\rangle$ and $|a^s\rangle$
which we may suppose the eigenstates
of some maximal observable A for
either particle with eigenvalues a^n and
a^s as indicated by the notation
for the states. By analogy with
the situation in classical physics
we might suppose that there are
four possibilities:

(1) Both particles are in the state $|a^n\rangle$

(2) Both particles are in the state $|a^s\rangle$

(3) Particle 1 is in state $|a^n\rangle$ and particle 2 in state $|a^s\rangle$

(4) Particle 1 is in state $|a^s\rangle$ and particle 2 in state $|a^n\rangle$.

All these four possible arrangements would be given equal weight in classical statistical mechanics

But in quantum statistics things are different.

Now in classical statistical mechanics arrangements 3 and 4 would be counted as distinct and given equal weight in assigning probabilities. But in quantum statistics, whether bosonic or fermionic, the arrangements 3 and 4 are counted as one and the same arrangement for the purposes of assigning weights. This is taken to show that the two arrangements are not only indistinguishable but are actually identical. But ontologically speaking these two arrangements are not identical if the two quantum particles are individuals. Hence the quantum particles cannot be individuals.

In passing we may note that this argument, which purports to show that quantum particles fall outside the scope of PII, since they are not individuals, is also sometimes invoked to show that

PII does apply to the states of affairs represented by the two arrangements 3 and 4. What about arrangements 1 and 2? Since the states this is where bosons differ from fermions. For bosonic particles 1 and 2 are allowed arrangements to be carried with equal weight as compared with the identified 3-cum-4 arrangement. For Fermionic particles further arrangements 1 and 2 are not permitted at all. This is the famous Pauli Exclusion Principle.

We return in the next section to discuss the significance of this difference from the point of view of PII. But first we need to explain what is wrong with the argument concerning the identity of the arrangements 3 and 4.

We begin by writing down the state vectors for the combined two-particle system corresponding to the arrangements 1, 2, 3 and 4. They are:

$$|a^r\rangle \otimes |a^r\rangle \qquad (1)$$

$$|a^s\rangle \otimes |a^s\rangle \qquad (2)$$

$$|a^r\rangle \otimes |a^s\rangle \qquad (3)$$

and
$$|a^s\rangle \otimes |a^r\rangle \qquad (4)$$

where we use the convention that in a tensor product of two states the left-hand member refers to particle 1 and the right-hand member to particle 2.

Now it is quite true that if the quantum particles are individuals then the states $\psi^{(3)}$ and $\psi^{(4)}$ are not identical. But the important point to notice is that these states are not the ones used in discussing quantum statistical mechanics. The relevant states for that purpose are as follows:

$$|a^2\rangle \otimes |a^1\rangle \qquad \text{---} \quad (5)$$

$$|a^s\rangle \otimes |a^s\rangle \qquad \text{---} \quad (6)$$

$$\frac{1}{\sqrt{2}}\left(|a^2\rangle \otimes |a^s\rangle + |a^s\rangle \otimes |a^1\rangle\right) \text{---} (7)$$

and $$\frac{1}{\sqrt{2}}\left(|a^2\rangle \otimes |a^s\rangle - |a^s\rangle \otimes |a^2\rangle\right) \text{-}(8)$$

The four states V$^{(5)}$, VI$^{(6)}$, VII$^{(7)}$ and VIII$^{(8)}$ are mutually orthogonal and span the same subspace as the states I$^{(1)}$, II$^{(2)}$, III$^{(3)}$ and IV$^{(4)}$, but they are chosen so that V$^{(5)}$, VI$^{(6)}$ and VII$^{(7)}$ are symmetric under exchange of particle labels (i.e. under exchange of left-hand and right-hand members of all tensor products) while VIII$^{(8)}$ is antisymmetric (changes sign) under the same operation. Note that V$^{(5)}$ and VI$^{(6)}$ are the same states as I$^{(1)}$ and II$^{(2)}$. The crucial difference is between the pairs III$^{(3)}$ and IV$^{(4)}$, and VII$^{(7)}$ and VIII$^{(8)}$. Now VII$^{(7)}$ is no more identical with VIII$^{(8)}$ than is III$^{(3)}$ with IV$^{(4)}$.

But for bosons the states are restricted to the three symmetric possibilities. That is why $V_{\text{III}}(8)$ gets eliminated from the counting procedures, not because it gets identified with $V_{\text{III}}(7)$.

Similarly for fermions the states are restricted to the antisymmetric possibilities.

But in the simple example this eliminates $V(5)$, $V_{\text{I}}(6)$ and $V_{\text{II}}(7)$, so $V_{\text{III}}(8)$ alone gets counted, but again not because it gets identified with $V_{\text{II}}(7)$.

To put the matter another way, states with the wrong symmetry get eliminated because they are not accessible to the joint quantum system, not because they do — there are no such states!

It should be remembered that for time-evolution under a symmetric Hamiltonian the symmetry character of a state cannot change with time. So no transitions can occur between the symmetric bosonic states and antisymmetric fermionic states.

The upshot of our argument is to show not that quantum particles must be individuals but rather that it is possible for them to be individuals, despite the peculiarities of quantum statistics.

It is quite true that in quantum field theory (QFT) particles are not regarded as individuals. They are jointly (quantized) excitations of a field. Particle labels do not enter into the discussion at all.

If our simple problem of counting the number of states for a two-particle system distributed over two one-particle states were transformed to quantum field theory, then for a boson (commuting) field there would be just three states corresponding to a double excitation of either state (mode) plus a single excitation of both states (modes). Similarly for a fermionic (anti commuting) field, there is only one state since double excitations are not allowed. So the quantum statistics comes out the way we want if to.

It is also true that there are strong arguments, for regarding the 'quantized excitation' view of quantum particles as the correct one. However for the purposes of this paper we continue to envisage the possibility of treating quantum particles as individuals and proceed to discuss whether they would or would not obey PII.

3. The Indistinguishability Postulate

What do mean by saying that two
quantum particles of the same species
(characterized by their intrinsic properties)
are indistinguishable?
In quantum mechanics this is expressed by
the Indistinguishability Postulate (I.P)

$$\langle P\phi | Q | P\phi \rangle = \langle \phi | Q | \phi \rangle \qquad (q)$$
$$\forall Q, \forall \phi$$

where ϕ is an arbitrary N-particle state
and Q a possible observable on the
N-fold tensor product space of states.
$|P\phi\rangle$ is by observation for $P|\phi\rangle$
where P is the unitary operator, which
is associated with an arbitrary permutation of
the particle labels.

(q) Says ~~effectively~~ that it is not
possible to tell by measuring
the expectation value of any
observable, whether the state of
the system is $|\phi\rangle$ or $|P\phi\rangle$

We notice that a sufficient condition
for (1) to hold is that $|P\phi\rangle = \pm |\phi\rangle$
for any self-adjoint operator on
the N-particle state-space.
Then interprets (1) as a restriction on
the possible states for the N particle
system, ~~restricting them~~ allowing just the
boson or fermion possibility.
(Note that the choice of sign needs only

exist, allowing for the possibility of so-called para-statistics intermediate in character between bosonic and fermionic behaviour.

But even in the two-particle case it should be added that the Razmoh and Segulay approach does not restrict the available State, only their accessibility in the way we described in the previous section. From now on we shall restrict the discussion mainly to the simple two-particle case.

Let Q now denote a possible observable (self-adjoint operator) on a single particle. Considered as possible observables on the joint system we have two possibilities $Q \otimes I$ for observing Q on particle 1 and $I \otimes Q$ for observing Q on particle 2. Denote $Q \otimes I$ by Q_1 and $I \otimes Q$ by Q_2. Now it says in the Razmoh-Segulay interpretation, that although Q_1 and Q_2 are self-adjoint operators on the Hilbert space for the joint system, they cannot separately be observed. The intuition here is that observing Q_1 or Q_2 would involve knowing explicitly which particle was which and this is impossible if the particles are indistinguishable.

But from the point of view to observing $P I I$, it seems clear that

and their associated 'actualizations'
probabilities:

we should not restrict the discussion to attributes which can actually be observed. ~~If we did this then by very definition of undergrowth(?) PII would automatically be violated~~ this would restrict the discussion to symmetric considerations such as $Q_1 + Q_2$. The ontological significance of PII can only be brought out by discussing whether particles 1 and 2 have the same physical attributes expressed by Q_1 and Q_2 *Shorter while recognizing that these attributes can never be observed!

This is the task we shall attempt in the next portion.

4. Quantum Individuals and the Identity of Indiscernibles.

We begin by discussing the case of fermions. It has been claimed in the literature that the Pauli Exclusion Principle, prohibiting two fermionic particles from being in the same quantum state, is a clear ramification of PII. What is being prohibited apparently, is that two fermions shall have both the same intrinsic properties of mass, spin, electric charge, etc. and the same state-dependent properties expressed by expectation values of all quantum-mechanical physical magnitudes. But look at the allowed state VIII (8). It is not true in such a state that each particle is present in a different state. Each particle clearly 'partakes' of both the states $|a\rangle$ and $|q\rangle$ in the superposition of product states expressed in VIII (8). So might it not appear that in the allowed state both particles also have the same state-dependent properties, which would contradict PII. Let us formulate these state-dependent properties in terms of physical magnitudes such as Q_1 and Q_2 pertaining to each particle separately as discussed in the preceding section. On orthodox interpretations of quantum mechanics the properties Q_1 and Q_2 must be interpreted not as possessed values, but as properties to yield specified 'actualization' results in accordance with the familiar statistical algorithm for computing the associated probabilities.

and remembering $\sum_{n} |q^n\rangle\langle q^n| = I,$

$\langle a^n|a^s\rangle = 0$ and $\langle a^n|a^{\mathfrak{a}}\rangle = \langle a^s|q^s\rangle = 1$

Denoting the fermion state $\frac{|\Psi\rangle}{\sqrt{N}}$ (8) by $|\Psi\rangle$
we shall be interested in ~~computing~~ evaluating
comparing both monodic properties of the form

$$\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha) \quad \text{and} \quad \text{Prob}^{|\Psi\rangle}(Q_2 = q^\alpha)$$

~~but also~~ and $\overset{also}{\wedge}$ relational properties of the form

$$\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha \mid Q_2 = q^\beta) \quad \text{and}$$

$$\text{Prob}^{|\Psi\rangle}(Q_2 = q^\alpha \mid Q_1 = q^\beta)$$

These quantities are ~~very~~ easily computed from

$$\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha \, \& \, Q_2 = q^\beta)$$

$$= \left| \langle q^\alpha | \langle q^\beta | \left(|\Psi\rangle \right) \right|^2$$

$$= \frac{1}{2} \left| \langle q^\alpha | a^r \rangle \right|^2 + \frac{1}{2} \left| \langle q \right.$$

$$= \frac{1}{2} \left| \langle q^\alpha | a^r \rangle \right|^2 \cdot \left| \langle q^\beta | a^s \rangle \right|^2$$

$$+ \frac{1}{2} \left| \langle q^\alpha | a^s \rangle \right|^2 \cdot \left| \langle q^\beta | a^r \rangle \right|^2$$

$$- \text{Re} \, \langle a^r | q^\alpha \rangle \langle q^\alpha | a^s \rangle \langle a^s | q^\beta \rangle \langle q^\beta | a^r \rangle \qquad \text{---(10)}$$

Summing this result over q^α & q^β to obtain
the marginal probabilities and considering

$$\sum_{\alpha} |q^\alpha\rangle\langle q^\alpha| = \sum_{\beta} |q^\beta\rangle = \sum_{n} |a_n\rangle\langle a_n| = 1$$

and $\langle a^r | a^s \rangle = 0$, yields immediately
the equality

$$\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha) = \text{Prob}^{|\Psi\rangle}(Q_2 = q^\alpha)$$

$$= \frac{1}{2} \left| \langle q^\alpha | a^r \rangle \right|^2 + \frac{1}{2} \left| \langle q^\alpha | a^s \rangle \right|^2 \qquad \text{---(11)}$$

Similarly we find

$$\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha \mid Q_2 = q^\beta)$$

$$= \text{Prob}^{|\Psi\rangle}(Q_2 = q^\alpha \mid Q_1 = q^\beta)$$

$$= \Big[|\langle q^\alpha | a^2 \rangle|^2 \cdot |\langle q^\beta | a^5 \rangle|^2$$

$$+ |\langle q^\alpha | a^5 \rangle|^2 \cdot |\langle q^\beta | a^2 \rangle|^2$$

$$- 2\,\text{Re}\,\langle a^2 | q^\alpha \rangle \langle q^\alpha | a^5 \rangle \langle a^5 | q^\beta \rangle \langle q^\beta | a^2 \rangle \Big]$$

$$\Big/ \Big[|\langle q^\beta | a^2 \rangle|^2 + |\langle q^\beta | a^5 \rangle|^2 \Big]$$

$$\qquad\qquad\qquad\qquad - - (12)$$

The significance of (11) and (12) is that the two fermions in the state (8) do in fact have the same monadic properties and the same relational properties one to another, so the weakest form of PII, which we can formulate which involves both monadic properties and relational properties, is violated.

There are a number of comments we want to make concerning this conclusion and the way it was derived.

(1) In classical physics the state-dependent properties of a particle are completely specified by the maximally specific state

description (location in phase space).

Here we can replace the question, "Do ~~individual~~ (particles) have the same state-dependent properties?" with the question "Do the two particles have the same maximally specific state description?"

If we try the same move in quantum mechanics we run into the problem that for a so-called 'entangled' state such as (8) there are no pure states which can be ascribed to the separate particles. (If there were such states the state of the combined system would be the tensor product of the states in question, but (8) is not of the form of a tensor product — it's a superposition of tensor products.)

Now two states in $\langle \Pi \rangle$ ~~ascribed~~ for the maximally specific states. So if we identified the relevant properties of the two particles with the pure states they are in, we would have to conclude that there is no answer to the question "Do they have the same properties?"

A corollary of this result is that insofar as we can speak of states for the separate particles at all we must speak of mixed states.[10] Indeed the relevant mixed states are the same for the two particles[11] — (Gyobelelle mechanics of the states ...York 1957. This is of course the mathematical content of the result (13) for the marginal probabilities.)

distributions for Q_1 and Q_2. But our analysis has gone beyond that involving the (improper) mixed states of the separate particles, by considering also the so-called 'relational' conditional probabilities[?] given in (12).

(2) There is another sort of relational property we might consider expressed by conforming

$$\text{Prob}\left(Q_2 = q^\alpha \mid Q_2 = q^\beta\right) = \delta_{\alpha\beta} \qquad (13)$$

with $\text{Prob}\left(Q_1 = q^\alpha \mid Q_2 = q^\beta\right)$ given by (12)

These relational properties of particle 1 to itself as compared with relations of particle 1 to particle 2 we reject as vindication of PII, for the same argument as we derived in section 1. [this property has already presupposed individuation of particle 1, and hence cannot be used to account for that individuation via PII. In the case of the two conditional probabilities calculated in (12) this argument does not apply; here we are calculating relations that exist between the particle and the other, and asking whether, does particle 1 exhibit the same to the other particle each particle exhibit the same relation to the other particle. This does not presuppose] for ruling out alternate trivialization of PII. The purported vindication of PII again defeats

or regarding (13) as a ~~monadic~~ monadic property ~~of particle~~ of particle 2 whereas it is in fact a relational property of particle 2 to itself which is also true as a relation of particle 1 to itself.

(3) If we write $\alpha = \beta$ in equation (12) then we indeed find for fermions we report

$$\text{Prob}\left(Q_1 = q^\alpha \mid Q_2 = q^\alpha\right) = 0$$

This shows that if a ~~measurement~~ actualization ~~yields~~ of Q_2 gives a certain value then the probability is zero probability that a ~~measurement~~ concurrent actualization of Q_1 will be yield the ~~same~~ same value. This is the real expression of the Exclusion principle, but has no bearing on $\int I I$, if we adhere to the attitude view that ~~measurement~~ actualizations do not correspond to actually existing possessed values.

(4) This brings us to our final comment. On hidden-variable reconstructions of QM, the circumstance demonstrated in point 3 alone would lead us to regard $\int I I$ as vindicated for fermions (assuming ~~at~~ ~~if faithful~~ ~~measurement~~ that ~~fermion~~ actualization results merely reveal pre-existing values)

New para

We now turn to the case of bosons.

It is often assumed that purported predictions of QII depends on consideration of states such as (3) & (6) where both particles ~~are inded~~ can indeed be attributed the same pure state.[13]

Denoting the state (5) by $|\phi\rangle$, for example ~~yields the~~ we can easily obtain the following results corresponding to (11) and (12):

$$\text{Prob}^{|\phi\rangle}(Q_1 = q^\alpha) = \text{Prob}^{|\phi\rangle}(Q_2 = q^\alpha)$$

$$= |\langle q^\alpha | a^2 \rangle|^2 \quad - - (11')$$

and $\quad \text{Prob}^{|\phi\rangle}(Q_1 = q^\alpha | Q_2 = q^\beta) = \text{Prob}^{|\phi\rangle}(Q_2 = q^\alpha | Q_1 = q^\beta)$

$$= |\langle q^\alpha | a^2 \rangle|^2 \quad - - (12')$$

So, as we might expect, both monadic and ~~relativistic~~ relational properties are the same for the two particles.

But it should be noted that this conclusion is also true for the state (7), where two different states are ~~involving~~ involved. So too are the results (11) and (12) alike with the minus sign in front of the 'interference' term in (12) replaced by a plus sign.

Finally, we make a brief comment on the case of parafermions.

Here there are event states for which the monadic properties of the two identical particles are not the same, but equally there

to be made for transpositions, since any permutation can be represented as a product of transpositions, so even permutations are always associated with the plus sign, the distinction between bosons and fermions only arises for odd permutations)

But Messiah and Greenberg (1964) pointed out that (1) should, on a more profound analysis, be interpreted not as a restriction on states, but as a restriction on the possible observables for the N-particle system. On this view (1) can easily be shown to imply $P^{-1} \hat{O} P = \hat{O}$ or $\hat{O} P = P \hat{O}$, so any permitted $\hat{O}$ must commute with any permutation P. This in turn implies that $\hat{O}$ must be a symmetric function of the particle labels.

The label permutations provide effectively a set of non-Abelian superselecting operators which can be used to resolve the state space into non-combining sectors associated with irreducible representations of the symmetric group S_N. For two particles there are only two irreducible representations of S_2, the symmetric provided by states which are symmetric or antisymmetric in the particle labels. So we are back to the boson and fermion possibilities but, with more than two particles, higher-dimensional representations of the symmetric group

are
~~there exist~~ possible parparticle states for which
TT is violated in the same way as for
bosons and fermions.)

As an example consider the following
normalized state for three paraboson
of order 2 :

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}\left(|a^{\wedge}\rangle|a^{\wedge}\rangle|a^{s}\rangle - |a^{s}\rangle|a^{\wedge}\rangle|a^{\wedge}\rangle\right) \quad (.14)$$

where $|a^{\wedge}\rangle$ and $|a^{s}\rangle$ are two different
two-particle states and triple tensor
products are written in the degrees
of particle labels 1, 2 and 3.

Denoting $Q_{\alpha}\otimes I \otimes I$ by Q_1, $I\otimes Q\otimes I$ by
Q_2 and $I\otimes I \otimes Q_{3}$ by Q_3, we

easily for the triple joint distribution

$$\text{Prob}^{|\Psi'\rangle}\left(Q_1 = q^{\alpha}, Q_2 = q^{\beta}, Q_3 = q^{\delta}\right)$$

$$= \left|\langle q^{\alpha}|\otimes\langle q^{\beta}|\otimes\langle q^{\delta}| \left(|\Psi'\rangle\right)\right|^2$$

$$= \frac{1}{2}\Big[\,|\langle q^{\alpha}|a^{\wedge}\rangle|^2 \cdot |\langle q^{\beta}|a^{\wedge}\rangle|^2 \cdot |\langle q^{\delta}|a^{s}\rangle|^2$$

$$+ |\langle q^{\alpha}|a^{s}\rangle|^2 \cdot |\langle q^{\beta}|a^{\wedge}\rangle|^2 \cdot |\langle q^{\delta}|a^{\wedge}\rangle|^2$$

$$- 2\,\mathrm{Re}\,\langle a^{\wedge}|q^{\alpha}\rangle\langle q^{\alpha}|a^{s}\rangle \cdot$$
$$\langle a^{\wedge}|q^{\beta}\rangle\langle q^{\beta}|a^{\wedge}\rangle$$
$$\langle a^{s}|q^{\delta}\rangle\langle q^{\delta}|a^{\wedge}\rangle\,\Big]$$

$$\qquad\qquad\qquad ---(15)$$

From (15) we find immediately the marginal distributions

$$\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha) = \text{Prob}^{|\Psi\rangle}(Q_3 = q^\alpha)$$

$$= \frac{1}{2}\left(|\langle q^\alpha|a^2\rangle|^2 + |\langle q^\alpha|a^5\rangle|^2\right) \quad -(16)$$

while

$$\text{Prob}^{|\Psi\rangle}(Q_2 = q^\alpha) = |\langle q^\alpha|a^n\rangle|^2 \quad -(17)$$

Thus particles 1 and 3 have the same monadic properties expressed by the marginal distributions but there differ from the monadic properties of particle 2.

Let us now show that particles 1 and 3 also have the same relational properties with respect to both the remaining particles.

We can easily find that

$$\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha \mid Q_3 = q^\beta) = \text{Prob}^{|\Psi\rangle}(Q_3 = q^\alpha \mid Q_1 = q^\beta)$$

$$= \frac{\left[|\langle q^\alpha|a^2\rangle|^2 \cdot |\langle q^\beta|a^5\rangle|^2 + |\langle q^\alpha|a^5\rangle|^2 \cdot |\langle q^\beta|a^2\rangle|^2 - 2\text{Re}\langle a^n|q^\alpha\rangle\langle q^\alpha|a^5\rangle\langle a^5|q^\beta\rangle\langle q^\beta|a^2\rangle\right]}{\left[|\langle q^\alpha|a^2\rangle|^2 + |\langle q^\alpha|a^5\rangle|^2\right]}$$

$$(18)$$

Furthermore

$$\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha \mid Q_2 = q^\beta)$$

$$= \text{Prob}^{|\Psi\rangle}(Q_3 = q^\alpha \mid Q_2 = q^\beta)$$

$$= \frac{1}{2}\left[\,|\langle q^\alpha | a^1\rangle|^2 + |\langle q^\alpha | a^5\rangle|^2\,\right]$$

and finally

$$\text{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha \mid Q_2 = q^\beta \,\&\, Q_3 = q^\delta)$$

$$= \text{Prob}^{|\Psi\rangle}(Q_3 = q^\alpha \mid Q_2 = q^\beta \,\&\, Q_1 = q^\delta)$$

$$= \Bigg[\,|\langle q^\alpha | a^1\rangle|^2 \cdot |\langle q^\beta | a^5\rangle|^2 \cdot |\langle q^\delta | a^1\rangle|^2$$
$$+ |\langle q^\alpha | a^5\rangle|^2 \cdot |\langle q^\beta | a^1\rangle|^2 \cdot |\langle q^\delta | a^1\rangle|^2$$
$$- 2\,\text{Re}\,\langle a^1 | q^\alpha\rangle\langle q^\alpha | a^5\rangle\langle a^5 | q^\beta\rangle\langle q^\beta | a^1\rangle$$
$$\langle a^1 | q^\delta\rangle\langle q^\delta | a^1\rangle\,\Bigg]$$

$$\Bigg/ |\langle q^\beta | a^1\rangle|^2 \left[\,|\langle q^\delta | a^1\rangle|^2 + |\langle q^\delta | a^5\rangle|^2\,\right]$$

These results show that SSS is violated for particle 1 and 3 in the state $|\pm\rangle$, even in its weakest form, that excludes all the relevant relational properties.

so there is a case where PII is violated
for particles 1 and 2

5 Conclusion

There are two main conclusions of
this paper. Firstly that indistinguishable
particles in QM can be treated as
individuals, but secondly, if they are
so treated, then, on the most plausible
reading of what constitutes a property
of a quantal particle, then even
the weakest form of PII including
both monadic and relational properties,
is violated both for bosons and
fermions, and indeed for higher-order
paraparticles.

It should be noted that if quantal
particles are individuals, then
their individuality must be conferred
by TI. STC is not in general
available in QM, since particles do not
move in well-defined trajectories.
so the question of spatio-temporal continuity
of trajectory does not arise. The only
exception to this is where the N-particles
states involve well-defined wave-packets,
which diffuse sufficiently slowly over time,
as needed to provide for the classical
limit of sufficiently massive particles.
But it is clear that in the case
of macroscopic bodies where STC can
used to label the worldlabel the bodies,

Erector
* assuming them to be maximally effective.

the STC criterion actually conflicts with
the TI understanding of the bare particle
elementary particles company the body.
To be strict every electron, further
1 the state of every other electron in
the universe according to the
anti-symmetrization requirement!

But notice, that under conditions where
the 'interference' terms in (10) can be
neglected, the state $|4\rangle$ behaves
like a proper mixture of states in
which particle 1 is in state $|a^r\rangle$ and
particle 2 in state $|a^s\rangle$ and the permuted
state in which particle 1 is in state $|a^s\rangle$
and particle 2 in state $|a^r\rangle$, with
equiprobable weights for the two different
states in the mixture. But under
these conditions the state (8) behaves like
an equiprobable mixture of states (3) and
(4). In other words, when 'interference' can
be neglected, we recover the same
possibilities for states as in classical
physics, where states (3) and (4) would anyway
be eliminated by TA.

But, of course, adequately stationary,
'interference' is never strictly absent.
That, after all, is what constitutes
the 'problem of measurement' in QM.
So the involvement of every electron
with the state of every other electron
in the universe, although negligible for
many practical purposes, remains an
uneliminated commitment of QM under
the interpretation where the particles
are treated as individuals.

If this sounds too bizarre to be acceptable, it provides another argument for preferring the treatment of indistinguishable particles along the lines provided by quantum field theory.

In this paper we have been concerned with conceptual possibilities rather than what it is most reasonable to believe about the ontological status of elementary particles.

The locus classicus is generally held to the Leibniz (196?) Book II chapter XXVII §1?. For an ~~influential~~ recent influential critical discussion see in particular Shingure (1976).

Note 12 ~~11~~

It should be stressed that those relational properties expressed by the conditional probabilities in no way supervene on the monadic monadic properties expressed by the marginal distributions. In the terminology of Teller (1986) they are ~~rather~~ inherent relations.

Note ?

There is much discussion in the literature as to whether a clear-cut distinction can be made between relational and monadic properties. For a ~~useful~~ comment see Hay [1984].

Notes → Ph.D. acknowledgment - See p.29.

2 †. There is no real consensus about Leibniz's own views on the status of PII. Frankfurt [1976] is an authoritative collection of critical essays that deal with this aspect (among many others) of Leibniz's philosophy. See in particular the contribution by Ishiguro entitled *

‡3 This terminology is due to Post [1963].

4 ‡. This interpretation of PII is mooted in Lucas [1984] p. 131.

8 ‡. The justification for assuming that the Hamiltonian observable must be a symmetric function of the particle labels will become apparent in Section 3.

5. In the physics literature such particles are often referred as `identical'. In our terminology this would mean they were one and the same particle! We shall use the term `indistinguishable', where in place of the physicist's `identical'.

7 6. See Redhead [1983] and [1986] for further discussion of the QFT approach.

7. We refer to 'actualization' rather 'measurement result', to emphasise the point that they are not observable. They will however, be produced by measurement interactions, but not in a way which makes them identifiable as contrasted with their particle label-permuted variants.

8. See Margenau [1944] and [1950] and Shadmi [1978]

9. 10. See D'Espagnat [1976] pp. 58-61 for a discussion of mixed states arising in this way. He calls them 'improper' mixtures.

10. See, for example, the discussion of PJI for bosons given by Cortes [1976], Barnett [1978], Ginsburg [1981] and Teller [1983].

Compare Hartle and Taylor [1969] for details of how to construct parastatistics states. The state (14) is obtained from their (2.5) by identifying two of the one-particle states.

A similar point is made in Van Fraassen [1984]. See also Margenau [1944] and [1950].

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†Past Some of the arguments in this paper offered in Ph.D.a thesis. Submitted by one of us (S.F.) in parteof fulfilment of the requirements of the University of Lord for the Ph.D. degree of the University of London, in 1984, entitled "Identity and Indistinguishability in Classical and Quantum Physics".

Please Insert at beginning of Notes

are always ~~other~~ possible paraparticle state where PJI is violated, in the ~~same~~ way as for bosons and fermions.

As an example consider the following normalized state for three paraparticles of order 2^{12}

$$|\psi'\rangle = \tfrac{1}{2}\left(|a^2\rangle \otimes |a^7\rangle \otimes |a^5\rangle + |a^5\rangle \otimes |a^7\rangle \otimes |a^2\rangle - |a^7\rangle \otimes |a^5\rangle \otimes |a^2\rangle - |a^2\rangle \otimes |a^5\rangle \otimes |a^7\rangle\right)$$

$$|\psi'\rangle = \tfrac{1}{\sqrt{2}}\left(|a^7\rangle \otimes |a^5\rangle \otimes |a^2\rangle - |a^5\rangle \otimes |a^2\rangle \otimes |a^7\rangle\right) \quad (14)$$

where $|a^2\rangle$, and $|a^5\rangle$ and $|a^7\rangle$ are ~~that~~ two different one-particle states and triple tensor products are written in the sequence of particle labels 1, 2 and 3.

Denoting $Q \otimes J \otimes J$ by Q_1 and $J \otimes Q \otimes J$ by Q_2 and $I \otimes J \otimes Q$ by Q_3, we easily find

$$\text{Prob}^{|\psi'\rangle}(Q_1 = q^\alpha) = \text{Prob}^{|\psi'\rangle}(Q_2 = q^\alpha) = \tfrac{1}{4}\left(|\langle q^\alpha|a^2\rangle|^2 + |\langle q^\alpha|a^5\rangle|^2 + 2|\langle q^\alpha|a^5\rangle|^2\right)$$
$$= \tfrac{1}{2}\left(|\langle q^\alpha|a^2\rangle|^2 + |\langle q^\alpha|a^5\rangle|^2\right) \qquad \qquad ---(15)$$

while
$$\text{Prob}^{|\psi'\rangle}(Q_3 = q^\alpha) = \tfrac{1}{2}\left(|\langle q^\alpha|a^5\rangle|^2 + |\langle q^\alpha|a^2\rangle|^2\right)$$
$$= |\langle q^\alpha|a^2\rangle|^2 \qquad \qquad - - - (16)$$

However if we ~~small~~ $a=5$, then the two marginal probabilities do become equal and we can easily see that the relative properties of particles with respect to both the remaining particles or else the same,